

Separation of Heterogeneous Solid – Fluid Mixtures.

VIII. Model for flowing through ideal layers of precipitate

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During phase separation from solid – fluid systems by filtration, the main process involved could be considered the fluid flowing through the precipitate which is usually approximated like a fine granular bed of particles already settled on the filtration medium (support). Dimensioning of filtration devices is usually based on experimental data to obtain filtration equation's parameters, but these constants are not related with pores' geometry. This paper presents an approach which takes into account a strictly definite geometry for the pores in the evolving precipitate, consisting in two types of models, one being purely geometric while the other is a modification of a "traditional" one, based on spherical shape (ψ) and void fraction (ϵ).

Keywords: filtration modeling, real geometry of precipitate pores, flowing through packed beds

Filtration of concentrate suspensions is an unit operation which separates fine divided solids from liquid suspension media; this operation usually follows the precipitation and precedes the draining and drying of the cake, the solid being the significant phase of the heterogeneous solid – fluid mixture. The separation potential applied to filtration is, most of the time, a gravitational pressure drop (Δp), realized either by vacuum the collecting space of the filtrate, or by super-atmospheric compression of the suspension space (if not both, or even by centrifugation). Precipitates can be either crystalline (and furthermore, incompressible) or amorphous conglomerates (which are compressible – it follows that an agglutination agent favors a faster filtration because the resistance of the precipitate to the liquid flowing decreases). The essential phenomenon in filtration process is the liquid flowing through the precipitate's pores, the velocity being a function of pores' dimensions (section and length) and also of the liquid viscosity (μ). Obviously, the gradually settling of the precipitate on the filtering material – canvas, sieve, other porous media, which act only as a support and could be considered having short pores, uniformly distributed and constant – smoothly decreases the filtrate flow, so a temperature increasing is beneficial, by lowering the liquid viscosity. As a rule, the flowing process through the precipitate layer (taking into account the support or not, according to its resistance) is described with equations needing empirical constants. In only some rare exceptions [1 ÷ 3], statistical characteristics of the precipitate – although vague, like void fraction (ϵ) or specific surface (σ) – are used to calculate the equivalent diameter ($d_{\text{ech,por}}$) for the pores in the granular bed. In this paper, an approach to the flowing process is made, assuming that solid particles are small, quasi-spherical, mono-disperse, so the precipitate's pores are capillaries with known forms and dimensions (but not at all cylindrical!).

Internal geometry of granular beds

The literature concerning flowing fluids through granular beds proposed a lot of models to calculate the pressure drop needed to realize a given velocity – one could quote equations given by **Darcy**, **Blake – Kozeny**, **Kozeny – Carman**, **Burke – Plummer**, **Ergun**, **Javoronkov – Aerov**,

Rose, and **Brownell** (probably the most sophisticated), all of them containing overall statistic characteristics of the layer (ϵ, σ) and mean values for non-spherical particles (d_p, ψ). Also, correction factors are used to adjust and fit equations against experimental data.

Two models, both of them based on internal geometry of the precipitate layer (and not mentioning ϵ and σ), are proposed further. They are purely geometric, taking into account uniform spherical particles in two extreme arrangements; also, a modified model using only ϵ and some coefficients derived with hydrodynamic considerations from the comparison with above geometrical model is presented. To describe the internal geometry of the precipitate layer, identical spherical particles, having a constant diameter (d_s), are considered to be settled in two extreme arrangements: a **compact** one, with regular tetrahedral symmetry, opposite to a **loose** one, with a cubic symmetry. The reference unit for the precipitate layer volume is a cubic cell, having the edge with an arbitrary length L_{cel}

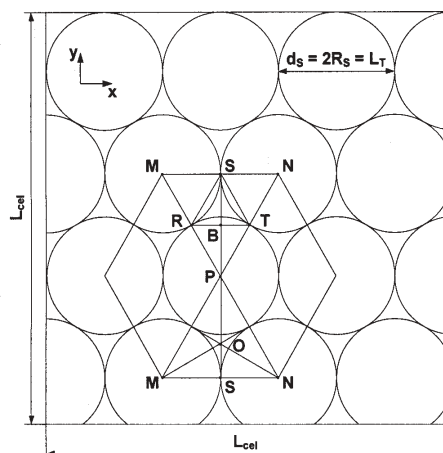


Fig. 1. The most compact layout of tangent spheres (d_s – diameter, R_s – radius) in a horizontal layer (L_{cel} – the edge of a cubic cell) with hexagonal symmetry.

The most compact layout of spherical particles

The basic 3-d cell of this layout consists of 4 adjacent spheres, their centers forming a regular tetrahedron. To

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see this, first the most compact layout in a single layer (2-d) should be pictured.

Every one front (or face) of the presumed tetrahedron is an equilateral triangle – for example, the one having corners denoted MNP, being centers of three coplanar and tangent spheres. This triangle is described by the side L_{tech} (MN) and the height h_{tech} (\overline{PS}). In addition, the symmetry of this most compact layout in 2-d yields to be hexagonal, so a “central” sphere is surrounded by other six tangent ones. The basic 2-d shapes are equilateral triangles or regular hexagons (fig. 1).

Now let's project the third dimension: besides on this layer, say horizontal, the “central” sphere is surrounded also by other six spheres, three on each side (above and below). Each group of three spheres – *i.e.*, their centers – forms neighbor layers (superior and inferior, respectively); the distance between these layers being h_T (the tetrahedron height, OQ in fig. 2).

From both figures 1 and 2 could be seen that three adjacent spheres from the horizontal plane, having centers in M, N, P, form what we call a “curvilinear equilateral triangle”, having as sides equal arches, with radius R_s and length L_c , further yielding to a perimeter equal with a half of the sphere great circle length. This is proved, among other results, from the following geometrical successive relations – see again figures 1 and 2:

$$\Delta MNP = \Delta MNQ = \Delta NPQ = \Delta MPQ$$

$$\overline{MN} = \overline{NP} = \overline{MP} = \overline{MQ} = \overline{NQ} = \overline{PQ} = L_x = L_T = 2 \cdot R_s = d_s$$

$$\overline{PS} = \overline{MT} = \overline{NR} = \overline{QR} = \overline{QS} = \overline{QT} = h_y$$

$$\overline{PS} = \sqrt{\overline{MP}^2 - \overline{MS}^2} = \sqrt{\overline{MP}^2 - (\overline{MN}/2)^2} = h_y = \sqrt{d_s^2 - d_s^2/4} = (\sqrt{3}/2) \cdot d_s$$

$$\overline{OP} = (2/3) \cdot \overline{PS} = \overline{OM} = \overline{ON} = (\sqrt{3}/3) \cdot d_s$$

$$\overline{OS} = (1/3) \cdot \overline{PS} = \overline{OR} = \overline{OT} = R_{tech} = (\sqrt{3}/6) \cdot d_s$$

$$\overline{OQ} = \sqrt{\overline{QS}^2 - \overline{OS}^2} = \sqrt{(3/4) \cdot d_s^2 - d_s^2/12} = h_z = h_r = (\sqrt{2}/3) \cdot d_s$$

$$\overline{OQ} = \sqrt{\overline{PQ}^2 - \overline{OP}^2} = \sqrt{d_s^2 - d_s^2/3} = h_z = h_r = (\sqrt{2}/3) \cdot d_s$$

$$\Delta MRS = \Delta NST = \Delta PRT = \Delta RST$$

$$\overline{RS} = \overline{RT} = \overline{ST} = \overline{MS} = \overline{MR} = \overline{PR} = \overline{PT} = \overline{NS} = \overline{NT} = L_{tech} = R_s = d_s / 2$$

$$\overline{CR} = \overline{AT} = \overline{BS} = \sqrt{\overline{RS}^2 - \overline{CS}^2} = \sqrt{\overline{RS}^2 - (\overline{ST}/2)^2} = \sqrt{d_s^2/4 - d_s^2/16} = h_{tech} = (\sqrt{3}/4) \cdot d_s$$

$$\overline{CR} = \overline{AT} = \overline{BS} = h_{tech} = h_y / 2 = \overline{NR} / 2 = \overline{MT} / 2 = \overline{PS} / 2 = \overline{CN} = \overline{AM} = \overline{BP} = (\sqrt{3}/4) \cdot d_s$$

A much closer examination of the channel (as a curvilinear triangle) formed between three adjacent spheres yields to figure 3.

One could further approximate and derive relations which arrive to the calculus of sides and surface of the flowing cross-section of the pore – as shown in figure 3, only the “relatively-free” area ($\Delta A'B'C'$) could be considered, while the narrow angular space (say, lamellar) is neglected, taking into account the obvious huge friction in those sections. Also, a more restrictive approach could consider the pore consisting only in the inner (inscribed) circle. First of all, it's obvious that $\Delta ABC < \Delta A'B'C'$; the calculus of all required dimensions follows:

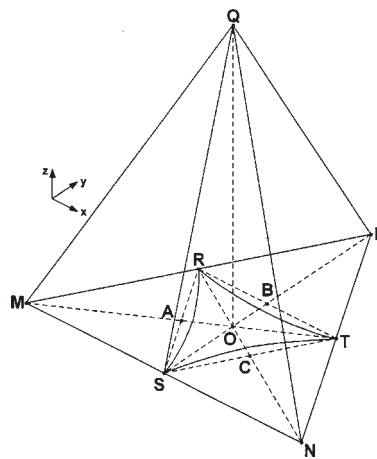


Fig.2. The 3-d basic cell of the most compact layout – a regular (equiangular) tetrahedron.

$$\overline{OD'} = \overline{OM} - R_s = (1/\sqrt{3} - 1/2) \cdot d_s$$

$$\overline{C'D'} = 3 \cdot \overline{OD'} = h_r = (\sqrt{3} - 3/2) \cdot d_s$$

$$\overline{A'B'} = L_r = (2/\sqrt{3}) \cdot h_r = (2 - \sqrt{3}) \cdot d_s$$

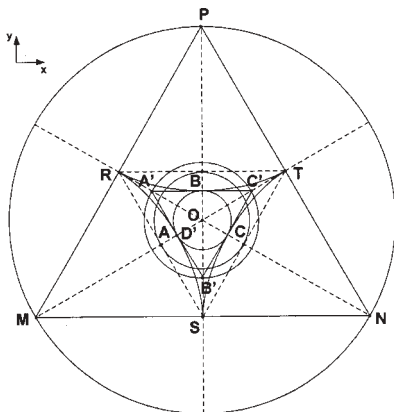


Fig. 3. A face of a tetrahedron with details of the flowing channel.

The number of pores in a horizontal section – say, xy – could be related to the perimeter of spheres' sections compared to the empty space (*i.e.*, the free flowing section) which is a sum of curvilinear triangles... and should be underlined that the number of pores is **twice** the number of spheres since their perimeter is a **half** of a great circle of a sphere:

$$\widehat{RS} = \widehat{ST} = \widehat{TR} = L_{ic} = \frac{\pi}{3} R_s = \frac{\pi}{6} d_s$$

$$\widehat{RS} + \widehat{ST} + \widehat{TR} = P_{ic} = \frac{\pi}{2} d_s = \pi \cdot R_s$$

$$n_{Sx} = L_{cel} / d_s$$

$$n_{Sy} = L_{cel} / h_y = (2 / \sqrt{3}) \cdot n_{Sx}$$

$$n_{Sxy} = n_{Sx} \cdot n_{Sy} = (2 / \sqrt{3}) \cdot n_{Sx}^2 = (2 / \sqrt{3}) \cdot (L_{cel} / d_s)^2$$

$$n_{Sz} = L_{cel} / h_z = (\sqrt{3} / 2) \cdot n_{Sx}$$

$$n_{Sxyz} = n_{Sx} \cdot n_{Sy} \cdot n_{Sz} = (2 / \sqrt{2}) \cdot n_{Sx}^3 = \sqrt{2} \cdot (L_{cel} / d_s)^3$$

$$V_{cel} = L_{cel}^3$$

$$\Sigma V_S = n_{Sxyz} \cdot \pi \cdot d_s^3 / 6 = \pi \cdot \sqrt{2} / 6 \cdot L_{cel}^3$$

$$\varepsilon_{strat} = (V_{cel} - \Sigma V_S) / V_{cel} = 1 - \pi \cdot \sqrt{2} / 6 = 0,25952...$$

The minimal cross-section of flowing (*i.e.*, the effective section of one channel / pore) could be estimated with respect to this section's form: either the curvilinear triangle RST, or the triangle $\Delta A'B'C'$, or even the circle having the radius OD' (inner circle, fig. 3); equivalent diameters are used to compare these forms:

$$d_{ech,ic} = 4 \cdot A_{ic} / P_{ic} = 4 \cdot (A_{\Delta MNP} - \pi \cdot R_s^2 / 2) / (\pi \cdot R_s) = 2 \cdot d_s \cdot (\sqrt{3} - \pi / 2) / \pi = (2 \cdot \sqrt{3} / \pi - 1) \cdot d_s$$

$$d_{ech,tr} = 4 \cdot A_{tr} / P_{tr} = 2 \cdot (\sqrt{3} - 3 / 2) \cdot d_s / 3 = (2 / \sqrt{3} - 1) \cdot d_s$$

$$d_{ech} = 2 \cdot \overline{OD'} = (2 / \sqrt{3} - 1) \cdot d_s \cong 0,1547 \cdot d_s$$

The pore is assumed further to be the inner circle since it has the same equivalent diameter with the equilateral

triangle – the curvilinear triangle could not be a real free flowing section while a certain agglutination takes place in those narrow (lamellar) spaces, among a huge friction.

The specific surface yields from:

$$A_S = \pi \cdot d_s^2 / 4$$

$$\Sigma A_S = n_{Sxy} \cdot A_S = \pi \cdot L_{cel}^2 / (2 \cdot \sqrt{3})$$

$$\sigma_S = A_S / V_S = \pi \cdot d_s^2 / (\pi \cdot d_s^3 / 6) = 6 / d_s$$

For the entire cubic cell, containing spherical particles:

$$n_{Sxyz} \cdot A_S / \Sigma V_S = 6 / d_s = \sigma_S \quad \sigma_{strat} = n_{Sxyz} \cdot A_S / V_{cel} = \sigma_S \cdot (1 - \varepsilon_{strat})$$

The number of pores is – according to above considerations – double with respect to the number of spheres, so:

$$n_{por,xy} = 2 \cdot n_{Sxy} = (4 / \sqrt{3}) \cdot (L_{cel} / d_s)^2$$

The apparent free section is given by the sum of all curvilinear triangles, or:

$$A_{cel} = L_{cel}^2; \quad \Sigma A_{ic} = A_{cel} - \Sigma A_S = L_{cel}^2 \cdot [1 - \pi / (2 \cdot \sqrt{3})]$$

while the effective flowing section takes into account the dimension of the considered pore:

$$\Sigma A_{por} = n_{por,xy} \cdot \pi \cdot d_{ech}^2 / 4 = L_{cel}^2 \cdot (\pi / \sqrt{3}) \cdot (2 / \sqrt{3} - 1)^2$$

Some ratios are useful to compare these geometries:

$$F_{ic} = \Sigma A_{ic} / L_{cel}^2 = 0,0931...$$

$$F_{por} = \Sigma A_{por} / L_{cel}^2 = 0,043408...$$

It follows that the relative effective flowing section is less than a half from the apparent (geometrical) one.

Usually, the equivalent diameter for packed beds is calculated with:

$$d_{eq} = 4 \cdot \varepsilon_{strat} / \sigma_{strat} = (2 / 3) \cdot d_s \cdot [6 / (\pi \cdot \sqrt{2}) - 1]$$

$$d_{eq} = 4 \cdot [\varepsilon_{strat} / (1 - \varepsilon_{strat})] / (d_s / 6) = (2 / 3) \cdot d_s \cdot [6 / (\pi \cdot \sqrt{2}) - 1]$$

but this is an approximation for the particle, not the channel! This are the major problems we want to solve in this approach: on one hand, from the geometrical point of view, the **equivalent** diameter is different from the **hydraulic** one – chemical engineers use to say “equivalent” instead “hydraulic”, in fact (!); on the other hand, this hydraulic diameter is used to further calculate **Re** number and estimate the flowing regime, but it's not always clear if the “diameter” stands for particles or for pores (when both categories are of the same scale, it may be confusing). To conclude, in the above described case – considered in the **same time** a packed (granular) bed and a porous media (!), the ratio between those equivalent diameters is

$$d_{eq} / d_{ech} = [4 / (\pi \cdot \sqrt{2}) - (2 / 3)] / (2 / \sqrt{3} - 1) \cong 1,51$$

The tortuosity of the considered pores (or how these channels are evolving through the bed, on the third dimension) is pictured in figure 4.

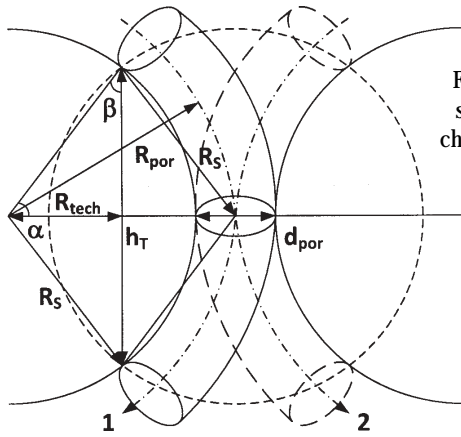


Fig. 4. Elements of simplified flowing channels, in different levels of depth.

The pores are approximated with curvilinear cylinders; two of them are shown in figure 4, allowing to estimate more dimensions:

$$\hat{\alpha} + \hat{\beta} = 90^\circ \Rightarrow R_{tech} = \sqrt{R_S^2 - (h_T/2)^2} = \sqrt{R_S^2 - 2 \cdot R_S^2/3} = R_S / \sqrt{3} = (\sqrt{3}/6) \cdot d_S$$

$$R_{por} = 2 \cdot R_{tech} = (2/\sqrt{3}) \cdot R_S = d_S / \sqrt{3}$$

$$\sin \alpha = (h_T/2) / R_S = \sqrt{2/3} \Rightarrow \alpha = 54.736^\circ \Rightarrow \beta = 35.264^\circ$$

$$L_{por} = (2 \cdot \alpha \cdot \pi / 180) \cdot R_{por} = [(\alpha \cdot \pi / 90) / \sqrt{3}] \cdot d_S \cong 1.103 \cdot d_S$$

$$L_{por} / h_{por} = 1.103 / (\sqrt{2/3}) \cong 1.351$$

i.e., the real length of the curved channel is with 35% bigger than the height of the considered layer (for example, L_{cel}). Some results are retained for future comparisons:

$$\varepsilon_C = 1 - \pi \cdot \sqrt{2} / 6 \quad (1)$$

$$\sigma_C = \sigma_S \cdot (1 - \varepsilon_C) = \pi \cdot \sqrt{2} / d_S \quad (2)$$

$$d_{ech,C} = (2/3) \cdot [6 / (\pi \cdot \sqrt{2}) - 1] \cdot d_S \quad (3)$$

$$d_{por,C} = (2/\sqrt{3} - 1) \cdot d_S \quad (4)$$

$$N_{por,A,C} = (4/\sqrt{3}) \cdot A / d_S^2 \quad (5)$$

$$\Sigma A_{por,C} = (\pi/\sqrt{3}) \cdot (2/\sqrt{3} - 1)^2 \cdot A \quad (6)$$

$$L_{por,C} \cong 1.351 \cdot \delta \quad (7)$$

The least compact layout of spherical particles

In both figures 5 and 6 are pictured geometrical elements for calculus of flowing in the other extreme layout of spherical particles of the same size (the least compact one).

$$\overline{MN} = \overline{NP} = \overline{PR} = \overline{RM} = 2 \cdot R_S = d_S$$

$$\overline{RN} = \overline{MP} = \sqrt{2} \cdot d_S$$

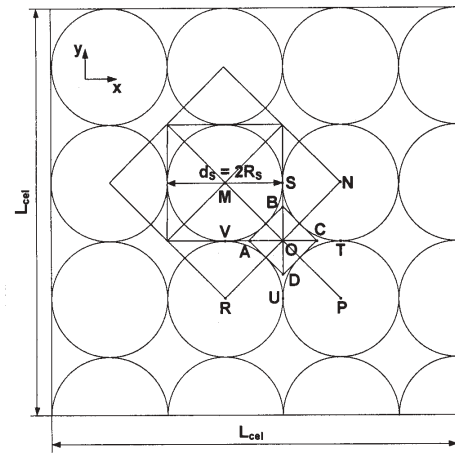


Fig. 5. The least compact layout of tangent spheres (d_S - diameter, R_S - radius) in a horizontal layer (L_{cel} - the edge of a cubic cell) with cubic symmetry.

$$n_{Sx} = n_{Sy} = n_{Sz} = L_{cel} / d_S$$

$$n_{Sxy} = n_{Sx} \cdot n_{Sy} = n_{Sxz} = n_{Syz} = (L_{cel} / d_S)^2$$

$$n_{Sxyz} = n_{Sx} \cdot n_{Sy} \cdot n_{Sz} = (L_{cel} / d_S)^3$$

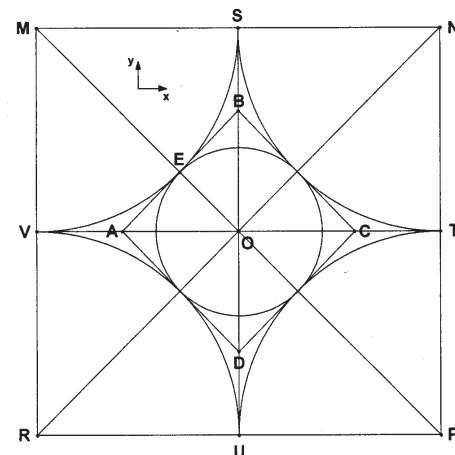


Fig. 6. A square (face of the cube) with details of the flowing channel

$$\overline{VS} = \overline{ST} = \overline{TU} = \overline{UV} = L_{pc} = (\pi/2) \cdot R_S = (\pi/4) \cdot d_S$$

$$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DA} = L_{pat} = \overline{RN} - 2 \cdot R_S = (\sqrt{2} - 1) \cdot d_S$$

$$\overline{AC} = \overline{BD} = \sqrt{2} \cdot L_{pat} = (2 - \sqrt{2}) \cdot d_S$$

$$\overline{OE} = L_{pat} / 2 = [(\sqrt{2} - 1) / 2] \cdot d_S$$

$$\overline{VS} + \overline{ST} + \overline{TU} + \overline{UV} = P_{pc} = 4 \cdot L_{pc} = \pi \cdot d_S$$

$$A_{pc} = \overline{MN}^2 - \pi \cdot d_S^2 / 4 = (1 - \pi/4) \cdot d_S^2$$

Then, similar assumptions could be made, concerning the crossing section (blocked and free) from figures 5 and 6, and furthermore, for the entire cubic cell:

$$A_S = \pi \cdot d_S^2 / 4; \Sigma A_S = n_{Sxy} \cdot A_S = (\pi/4) \cdot L_{cel}^2$$

$$A_{cel} = L_{cel}^2; \Sigma A_{por} = A_{cel} - \Sigma A_S = (1 - \pi/4) \cdot L_{cel}^2$$

$$V_S = \pi \cdot d_S^3 / 6; \Sigma V_S = n_{Sxyz} \cdot V_S = (\pi / 6) \cdot L_{cel}^3$$

$$V_{cel} = L_{cel}^3; \Sigma V_{por} = V_{cel} - \Sigma V_S = (1 - \pi / 6) \cdot L_{cel}^3$$

$$\sigma_{strat} = n_{Sxyz} \cdot A_S / V_{cel} = \sigma_S \cdot (1 - \varepsilon_{strat})$$

$$\varepsilon_{strat} = \Sigma V_{por} / V_{cel} = 1 - \pi / 6 = 0.4764...$$

Next, the equivalent diameter of a flowing channel (pore), assuming all three forms - curved square, square, inner circle (fig. 6) - follows:

$$d_{ech,pc} = 4 \cdot A_{pc} / P_{pc} = [(4 - \pi) / \pi] \cdot d_S$$

$$d_{ech,pat} = 4 \cdot A_{pat} / P_{pat} = 4 \cdot L_{pat}^2 / 4 \cdot L_{pat} = L_{pat} = (\sqrt{2} - 1) \cdot d_S$$

$$d_{ech} = 2 \cdot \overline{OE} = L_{pat} = (\sqrt{2} - 1) \cdot d_S$$

As previously, let's assume that only the inner circle is the effective flowing area of the pore, since all narrow lamellar sections could be considered like potential accumulation regions (and less flowing sections) - anyway, the square and the inner circle have the same equivalent diameter. In depth, every channel (pore) is straight, although it has a variable section (fig. 6 pictures the effective section, with the minimum area, while the maximum area is the whole surface of NMPPR). A comparison between the minimum and the maximum section yields to, according to the considered form:

$$F_{por} = A_{por,min} / A_{por,max} = (\pi \cdot d_{ech,pat}^2 / 4) / d_S^2 = \pi \cdot (\sqrt{2} - 1)^2 / 4 = 0.13475...$$

$$F_{pc} = A_{pc} / A_{por,max} = 1 - \pi / 4 = 0.2146...$$

$$F_{pc} / F_{por} = (4 - \pi) / [\pi \cdot (\sqrt{2} - 1)^2] = 1.5926...$$

The value of the last ratio confirm the reasonability of assuming the inner circle like the active (or effective) pore; nevertheless, neglected sections are - more or less - stagnant; on the other hand, repeated communication between neighbor channels is not considered, assuming the sideways is not important - in fact, it's impossible to evaluate (without an experiment using a tracer) the contribution of non-vertical directions (flowing against granular beds is an alternative for mixing fluids, as well).

Again, some of above results are retained for comparisons:

$$\varepsilon_a = 1 - \pi / 6 \quad (8)$$

$$\sigma_a = \sigma_S \cdot (1 - \varepsilon_a) = \pi / d_S \quad (9)$$

$$d_{ech,a} = [(4 / \pi) - (2 / 3)] \cdot d_S \quad (10)$$

$$d_{por,a} = (\sqrt{2} - 1) \cdot d_S \quad (11)$$

$$N_{por,A,a} = A / d_S^2 \quad (12)$$

$$\Sigma A_{por,a} = (\pi / 4) \cdot (\sqrt{2} - 1)^2 \cdot A \quad (13)$$

$$L_{por,a} = \delta \quad (14)$$

Laminar flow through granular beds

Theory of filtration yields to calculate the volumetric flow, G_v , of filtrate which goes through the resistance of empty filter, $R_{f,0}$ (or support) and also (except for the very first moment) the resistance of the already deposited precipitate, $R_{f,pr}$. The process takes place because of a pressure drop Δp - using the well-known electric analogy, the potential difference responsible for the "flowing" of electric charges, $i.e.$, the current. The simplest filtration model purely neglect the resistance of the support and consider only the variable resistance of the growing layer of precipitate; furthermore, for a very short (but still finite) period in time, the flowing is assumed to be stationary. We wish to compare here the classic approach, using ε_a and σ_a and assuming a flowing through quasi-cylindrical pores, with above described geometries consisting in ordered layers of identical and non-deformable spheres (in both layouts, compact and loose) - in fact, with flowing through pores of definite geometries. The only condition is: dimension of spheres ($i.e.$, diameter) should ensure a certain capillarity for the pores, so the flowing regime of an incompressible and Newtonian fluid could be assumed laminar.

Geometrical model

Assuming cylindrical and vertical pores, from the customization of **Navier - Stokes** equation, the well-known **Hagen - Poiseuille** [4] formulas for mean velocity, and volumetric flow, respectively, yields:

$$v = (\Delta p_{tot} \cdot d_{por}^2) / (32 \cdot \eta \cdot \delta_{pr}) \quad (15)$$

$$G_{v,por} = v \cdot A_{por} = (\pi \cdot \Delta p_{tot} \cdot d_{por}^4) / (128 \cdot \eta \cdot \delta_{pr}) \quad (16)$$

in which the total pressure drop could be assumed equal with the external one, neglecting the contribution of the above liquid layer ($i.e.$, gravity) and the friction inside; the pore's length is equal to the thickness of the precipitate, at a certain time. To compare the results against our models of the precipitate layer (either compact or loose), one should substitute the appropriate pore's diameter ($i.e.$, the hydraulic one) from relations (4) / (11) and pore's length (eventually, bigger than precipitate's thickness, because of the tortuosity, as shown) from relations (7) / (14), respectively, taking also into account the number of pores from relations (5) / (12). It follows:

$$G_{v,c} = v_c \cdot \Sigma A_{por,c} = \frac{\Delta p_{tot} \cdot d_{por,c}^2}{32 \cdot \eta \cdot L_{por,c}} \cdot \frac{\pi}{\sqrt{3}} \cdot \left(\frac{2}{\sqrt{3}} - 1\right)^2 \cdot A \quad (17)$$

$$G_{v,a} = v_a \cdot \Sigma A_{por,a} = \frac{\Delta p_{tot} \cdot d_{por,a}^2}{32 \cdot \eta \cdot L_{por,a}} \cdot \frac{\pi}{4} \cdot (\sqrt{2} - 1)^2 \cdot A \quad (18)$$

Traditional model

Usually [3,4], flowing through a granular (packed) bed is analyzed using an equivalent diameter for particles, $d_{ech,p}$ or d_v , meaning the diameter of a fictive sphere, having the same volume like the considered particle; also, the sphericity ψ is calculated, like the ratio between the equivalent sphere and the particle itself, respectively:

$$d_v = \sqrt[3]{6 \cdot V_p / \pi} \quad (19)$$

$$\psi = A_s / A_p = \pi \cdot d_v^2 / A_p = \sqrt{36 \cdot \pi \cdot V_p^{2/3}} / A_p \quad (20)$$

The problem in above relations are the dimensions for a "mean" particle, difficult to estimate (especially A_p). In

addition, while in relation (20) ψ is related with squared d_v , in other sources [5] is related with d_v only:

$$A_p / V_p = 6 / (\psi \cdot d_v) \quad (21)$$

Differences are not so important, since ψ is almost 1 in most of the cases (cubes, cylinders with unity invariant, and even sand). Specific surface and also the equivalent diameter of void space (!) are related with the void fraction ϵ :

$$\sigma_{strat} = \sigma_p \cdot (1 - \epsilon) / \psi = 6 \cdot (1 - \epsilon) / (\psi \cdot d_v) \quad (22)$$

$$d_{ech,por} = 4 \cdot \epsilon / \sigma_{strat} = (2/3) \cdot (\psi \cdot \epsilon \cdot d_v) / (1 - \epsilon) \quad (23)$$

There are two ways to estimate the volumetric flow: either using the real velocity and the real free section, or the fictive velocity - related to the entire surface of the filter (obviously, the same value is expected):

$$G_V = v_0 \cdot A_f = v_g \cdot \Sigma A_{por} \quad (24)$$

$$v_0 = v_g \cdot \Sigma A_{por} / A_f = v_g \cdot \delta_{pr} \cdot \Sigma A_{por} / V_{pr} = v_g \cdot \Sigma V_{por} / V_{pr} = v_g \cdot \epsilon \quad (25)$$

$$\begin{aligned} G_{V,g} = v_0 \cdot A_f = v_g \cdot \epsilon \cdot A_f &= \frac{\Delta p_{tot} \cdot d_{ech,por}^2}{32 \cdot \eta \cdot \delta} \cdot \epsilon \cdot A_f = \\ &= \frac{\Delta p_{tot} \cdot d_v^2}{72 \cdot \eta \cdot \delta} \cdot \frac{\psi^2 \cdot \epsilon^3}{(1 - \epsilon)^2} \cdot A_f \end{aligned} \quad (26)$$

Because, for a constant pressure gradient, the velocity given by (26) is considerably larger than the experiment, the constant 72 was empirically adjusted (with a factor of 25/12) to yield the **Blake - Kozeny** [2] relation:

$$\Delta p_{tot} = 150 \cdot \frac{v_0 \cdot \eta \cdot \delta}{d_v^2} \cdot \frac{(1 - \epsilon)^2}{\epsilon^3} \quad (27)$$

written for spherical particles (*i.e.*, $\psi = 1$).

The correlation (27) is valid only under some restrictions [2,4]:

$$\epsilon < 0.5; \quad Re_{strat} = \frac{d_s \cdot v_g \cdot \rho}{\eta} \cdot \frac{\epsilon}{1 - \epsilon} < 10 \quad (28)$$

and was included in relation **Ergun** for the laminar regime.

This correction (more than double!) is not justified by the tortuosity of the pores since previously was shown that increasing length for the most compact (!) layout is given by a 1.35 factor - see relation (7). Furthermore, other sources recommend a bigger numerical factor in (27), like a value in the domain (150 ÷ 180) - relation **Kozeny - Carman**, or even 200 [4]. For example, this constant is 180 in the formula given by **Kozeny** [1] for the hydrodynamic resistance of a granular layer of precipitate, consisting in spherical particles:

$$r_{pr} = \frac{180}{d_s^2} \cdot \frac{(1 - \epsilon)^2}{\epsilon^3} \quad (29)$$

Equations (15) and (27) could be translated into criterial forms:

$$\left(\frac{\Delta p_{tot}}{\rho \cdot v^2} \right) \cdot \frac{d_{por}}{L_{por}} \cdot \left(\frac{v \cdot \rho \cdot d_{por}}{\eta} \right) = Eu_{tot} \cdot \frac{1}{inv_{cil}} \cdot Re = 32; \quad Re < 2300 \quad (30)$$

$$\left(\frac{\Delta p_{tot}}{\rho \cdot v_0^2} \right) \cdot \left(\frac{d_s}{\delta} \cdot \frac{\epsilon^3}{1 - \epsilon} \right) \cdot \left(\frac{v_0 \cdot d_s \cdot \rho}{\eta} \cdot \frac{1}{1 - \epsilon} \right) =$$

$$= Eu_{strat} \cdot \frac{1}{inv_{strat}} \cdot Re_{strat} = 150 \div 180; \quad \epsilon < 0.5; \quad Re_{strat} < 10 \quad (31)$$

According with former assumption for " p_{tot} ,"

$$Eu_{tot} = Eu + Fr; \quad Fr = g \cdot h_{sus} / v^2 \quad (32)$$

in which **Fr** could be zero for horizontal filtration or negligible for vertical one (if its contribution is not important). Relation (32) is also valid for **Eu_{strat}**.

Comparison of models

It follows the reconciliation between two models of laminar flowing through interstitial pores in a layer of identical (an spherical) particles - the so-called "traditional", based on overall characteristics of the granular bed, like ϵ and σ , and eventually the form factor ψ , which needs correction factors bigger than 2 to fit against experimental data, and a purely geometrical one, based on idealized particles, consisting of two extreme types of layout (compact and loose). Several parameters could be explored: velocities, diameters, **Re** numbers, volumetric flow...

- first, a comparison between v_g from (26) for both cases of layout vs. velocity from (15), also in both cases; it yields, for the compact layout:

$$\begin{aligned} \frac{v_c}{v_{g,c}} &= \frac{\Delta p_{tot} \cdot d_s^2}{32 \cdot \eta \cdot L_{por,c}} \cdot \left(\frac{2}{\sqrt{3}} - 1 \right)^2 \cdot \frac{72 \cdot \eta \cdot L_g}{\Delta p_{tot} \cdot d_s^2} \cdot \left(\frac{1 - \epsilon_c}{\epsilon_c} \right)^2 = \\ &= \frac{72}{32} \cdot \left(\frac{2}{\sqrt{3}} - 1 \right)^2 \cdot \left(\frac{\pi \cdot \sqrt{2}}{6 - \pi \cdot \sqrt{2}} \right)^2 \cdot \frac{L_g}{L_{por,c}} \end{aligned}$$

If the length of the pores is assumed to be the same (*i.e.*, the same tortuosity), the equality of velocities claims to adjust the factor of 72 to 164.24, which approximate the arithmetic mean between 150 and 180, most recommended factors in literature [2 ÷ 6]. In conclusion, the ratio of the adjustment is around 2.28 or (3/2)² - and not 25/12; for the loose layout:

$$\begin{aligned} \frac{v_a}{v_{g,a}} &= \frac{\Delta p_{tot} \cdot d_s^2}{32 \cdot \eta \cdot L_{por,a}} \cdot (\sqrt{2} - 1)^2 \cdot \frac{72 \cdot \eta \cdot L_g}{\Delta p_{tot} \cdot d_s^2} \cdot \left(\frac{1 - \epsilon_a}{\epsilon_a} \right)^2 = \\ &= \frac{72}{32} \cdot (\sqrt{2} - 1)^2 \cdot \left(\frac{\pi}{6 - \pi} \right)^2 \cdot \frac{L_g}{L_{por,a}} \end{aligned}$$

Once again, the pores' length being the same, to have the same velocities yields to adjust the coefficient 72 to 154.5 (a little different from the arithmetic mean of 150 and 180), *i.e.*, the ratio should be 0.4663 or (2/3)². To conclude this, a real layout is somewhere between those limits; from given tables in literature for packed beds, it is obvious that when diameters are little, the void fraction is little (toward ϵ_c) while if diameters are bigger, the void fraction trend is to ϵ_a :

- a comparison between **Re** and **Re_{strat}** starts with the well-known definition, together with the assumption of the same velocity in both models (and the same fluid):

$$\frac{Re_{strat}}{Re} = \frac{v_g \cdot d_s}{v \cdot d_{por}} \cdot \frac{\epsilon}{1 - \epsilon} \approx \frac{d_s}{d_{por}} \cdot \frac{\epsilon}{1 - \epsilon}$$

For both extreme layouts one could calculate the ratio:

$$\frac{Re_{strat,c}}{Re_c} = \frac{d_s}{d_c} \cdot \frac{\epsilon_c}{1 - \epsilon_c} = \frac{3}{(2 \cdot \sqrt{3} - 3)} \cdot \left(\frac{3 \cdot \sqrt{2}}{\pi} - 1 \right) = 2.2655 \approx (3/2)^2$$

$$\frac{Re_{strat,a}}{Re_a} = \frac{d_s}{d_a} \cdot \frac{\epsilon_a}{1 - \epsilon_a} = \frac{1}{(\sqrt{2} - 1)} \cdot \frac{6 - \pi}{\pi} = 2.1966 \approx (3/2)^2$$

This result is remarkable – it states for a relative independence of layout and the flowing regime; of course, taking into account restrictions from (30) and (31);

- comparison between volumetric flows, G_v and G_{vg} requires the effective flowing area; assuming the calculus of pores' area in a cross-section includes also some stagnant zones, it follows that $\Sigma A_{por} = \kappa \cdot A_f \neq \varepsilon \cdot A_f$; replacing formulas for both layouts, we arrive to:

$$\kappa_c = \frac{4}{\sqrt{3}} \cdot \frac{A_f}{d_s^2} \cdot \frac{\pi \cdot d_c^2}{4 \cdot A_f} = 0.04314 \approx \frac{2}{3} \cdot \varepsilon_c^2$$

$$\kappa_c = \frac{A_f}{d_s^2} \cdot \frac{\pi \cdot d_a^2}{4 \cdot A_f} = 0.1348 \approx \frac{2}{3} \cdot \varepsilon_a^2$$

The modified traditional model

For a better consistency with the geometrical model, we used some modifications against the traditional model, as follows:

$$d_g = (2/3) \cdot d_{ech} = (2/3)^2 \cdot [\varepsilon / (1-\varepsilon)] \cdot d_s \quad (33)$$

$$v_g = \frac{\Delta p_{tot}}{32 \cdot \eta} \cdot \frac{d_g^2}{L_g} = \frac{\Delta p_{tot}}{32 \cdot \eta} \cdot \frac{d_s^2}{L_g} \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right)^2 \quad (34)$$

$$Re_g = \frac{v_g \cdot d_g \cdot \rho}{\eta} = \frac{\Delta p_{tot} \cdot \rho}{32 \cdot \eta} \cdot \frac{d_s^3}{L_g} \cdot \left(\frac{2}{3}\right)^6 \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right)^3 \quad (35)$$

$$G_{vg} = v_g \cdot (2/3) \cdot \varepsilon^2 \cdot A_f \quad (36)$$

All these could be sustained by all above estimations – σ is much more related to the wetted surface of particles and not the flowing section; inserting empirical corrections like various numerical coefficients is not elegant; moreover, if they are different, according to the layout, they are pretty confusing; relations (33÷36), using only ε , could be easily adjusted for different form factors; the only numerical correction introduced was $(2/3)^n$, already obtained from various theoretical considerations, shown here; in this manner, the only parameter to be obtained from experiments is ε , by far the most accessible. Even recent works [7÷9] still needs several parameters for modeling.

An exhaustive comparison between these models, geometrical and traditional (but modified) is shown in Table 1, containing values obtained for modeling the laminar flow of water at 20°C, against granular beds of spherical particles, of several diameters.

The deviations (between columns of the same parameter, obtained from different approaches) are more important for values of loose layout, which is less probable for a smaller d_s . They are bigger when the value is constructed using more constitutive quantities, but they have a maximum of 7.5% – it seems that they are much more accurate with respect to other references.

Conclusions

The geometrical model, with two limit layouts, containing identical and spherical particles, allows a precise calculus of the pores' geometry and further quantities in filtration's modeling. Taking into account the non-sphericity and also the non-ideal settling, a traditional model was modified to depend only on the void fraction ε , by far the most accessible parameter by experiment. Both models are giving consistent (and confident!) results; in a future work, they will be used for various particle types, other than spherical [10].

Table 1
NUMERICAL COMPARISON BETWEEN VALUES OBTAINED FROM BOTH MODELS, FOR WATER AT 20°C
($\rho = 10^3 \text{ kg/m}^3$; $\eta = 1 \text{ cP}$; $\Delta p = 750 \text{ Torr}$; $\delta = 1 \text{ cm}$; $A = 1 \text{ m}^2$)

d_s	d_{por} (4)	d_{por} (1.1)	ε_c	ε_a	d_{gc} (33)	d_{ga} (33)	V_c (15)	V_a (15)	V_{gc} (34)	V_{ga} (34)	Re_c	Re_a	Re_{gc} (35)	Re_{ga} (35)	N_{por}/A_f (5)	N_{por}/A_f (12)	G_v (17)	G_v (18)	G_{vgc} (36)	G_{vga} (36)
0.1	0.01547	0.04142	0.25952	0.4764	0.01558	0.04044	0.0055	0.0536	0.0056	0.0511	0.09	2.22	0.09	2.07	2.31E+08	1.00E+08	0.0002	0.0072	0.0003	0.0077
0.5	0.07735	0.20711	0.25952	0.4764	0.07788	0.20219	0.1384	0.3403	0.1403	1.2774	10.70	277.58	10.93	258.28	9.24E+06	4.00E+06	0.0060	0.1806	0.0063	0.1933
1	0.15470	0.41421	0.25952	0.4764	0.15577	0.40438	0.5535	5.3611	0.5612	5.1096	85.63	2220.65	87.41	2066.21	2.31E+06	1.00E+06	0.0240	0.7224	0.0252	0.7731
1.5	0.23205	0.62132	0.25952	0.4764	0.23365	0.60657	1.2454	12.0625	1.2627	11.4966	289.00	7494.68	295.02	6973.47	1.03E+06	4.44E+05	0.0541	---	0.0567	---
2	0.30940	0.82843	0.25952	0.4764	0.31153	0.80876	2.2141	21.4445	2.2447	20.4383	685.04	17765.18	699.30	16529.72	5.77E+05	2.50E+05	0.0961	---	0.1008	---
2.5	0.38675	1.03553	0.25952	0.4764	0.38942	1.01095	3.4595	33.5070	3.5074	31.9349	1337.97	34697.61	1365.83	32284.60	3.70E+05	1.60E+05	0.1502	---	0.1575	---

Note that in the shaded areas the condition of a laminar regime is no longer fulfilled.

Nomenclature

A – surface, area (m^2);
d – diameter (m);
Eu – **Euler** number;
F – fraction;
Fr – **Froude** number;
 G_v – volumetric flow (m^3 / s);
g – gravity ($9.81 m / s^2$);
h – height (m);
inv – invariant;
L – length (m);
n, N – number;
P – (wetted) perimeter (m);
p – pressure (N / m^2);
R – radius (m);
 R_f – resistance to filtration (m / s);
r – hydrodynamic resistance of a layer (m^2);
Re – **Reynolds** number;
V – volume (m^3);
v – velocity (m / s);

Greeks

α , ω – angles;
 Δ – difference, triangle;
 δ – thickness (m);
 ε – void fraction (m^3 / m^3);
 κ – effectiveness factor;
 μ – dynamic viscosity ($kg / m \cdot s$);
 π – invariant of circle;
 ρ – density (kg / m^3);
 Σ – summation;
 σ – specific surface (m^2 / m^3);
 ψ – sphericity.

Subscripts

a – loose;
C – compact;
cel – cell;
cil – cylinder;
eq, ech – equivalent (hydraulic);
f – filter;

g – void;
min – minimum;
max – maximum;
p – particle;
pat – square;
pc – curved square;
por – pore (channel, capillary);
pr – precipitate;
S – sphere;
strat – layer;
susp – suspension;
T – tetrahedron;
tc – curved triangle;
tech – equilateral triangle;
tot – total, overall;
tr – triangle;
V – volume;
x, y, z – Cartesian (horizontal, depth, vertical);
0 – fictive or related to support.

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